Chapter
Three Dimensional Geometry

Topic-1: Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1 MCQs with One Correct Answer

1. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals [2007 (S), 2010]

(a) 45° (b) 60° (c) 75° (d) 30°

2. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals [2004]

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Topic-2: Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1 MCQs with One Correct Answer

1. Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \in \{0,1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1, \ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1, \ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is [Adv. 2023]

(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{1}{\sqrt{8}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{12}}$

2. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is [2004S]

(a) 3/2 (b) 9/2 (c) −2/9 (d) −3/2

6 MCQs with One or More than One Correct Answer

3. Three lines $L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R$

$L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in R$ and

$L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear? [Adv. 2019]

(a) $\hat{k} - \frac{1}{2}\hat{j}$ (b) $\hat{k}$ (c) $\hat{k} + \hat{j}$ (d) $\hat{k} + \frac{1}{2}\hat{j}$

4. Let L_1 and L_2 denote the lines

$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$

and $\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3 ? [Adv. 2019]

(a) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(b) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(c) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(d) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

5. From a point $P(\lambda, \lambda, \lambda)$, perpendicular PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/(are) [Adv. 2014]

(a) $\sqrt{2}$ (b) 1 (c) -1 (d) $-\sqrt{2}$

6. A line l passing through the origin is perpendicular to the lines $l_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}, \quad -\infty < t < \infty$

$l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, \quad -\infty < s < \infty$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are) [Adv. 2013]

(a) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (b) $(-1, -1, 0)$

(c) $(1, 1, 1)$ (d) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

7 Match the Following

7. Let $\gamma \in \mathbb{R}$ be such that the lines

$L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$

intersect. Let R_1 be the point of intersection of L_1 and L_2. Let $O = (0, 0, 0)$, and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2. Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) γ equals	(1) $-\hat{i} - \hat{j} + \hat{k}$
(Q) A possible choice for $\hat{n}$ is	(2) $\sqrt{\frac{3}{2}}$
(R) $\overrightarrow{OR_1}$ equals	(3) 1
(S) A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4) $\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
	(5) $\sqrt{\frac{2}{3}}$

The correct option is [Adv. 2024]

(a) (P)→(3) (Q)→(4) (R)→(1) (S)→(2)
(b) (P)→(5) (Q)→(4) (R)→(1) (S)→(2)
(c) (P)→(3) (Q)→(4) (R)→(1) (S)→(5)
(d) (P)→(3) (Q)→(1) (R)→(4) (S)→(5)

8. Match the statement in **Column-I** with the values in **Column-II** [2010]

Column – I	Column – II
(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(p) -4
(B) The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0

(C) Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b} = 0$.
$(\vec{b}-\vec{a}).(\vec{b}+\vec{c}) = 0$ and $2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$.

If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are (r) 4

(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$

and $f(x) = \sin\left(\frac{9x}{2}\right) \Big/ \sin\left(\frac{x}{2}\right)$ for $x \neq 0$ (s) 5

The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is (t) 6

Topic-3: Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1 MCQs with One Correct Answer

1. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is **[2012]**

   (a) $5x - 11y + z = 17$ (b) $\sqrt{2}x + y = 3\sqrt{2} - 1$
   (c) $x + y + z = \sqrt{3}$ (d) $x - \sqrt{2}y = 1 - \sqrt{2}$

2. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is **[2012]**

   (a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

3. If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is **[2010]**

   (a) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (b) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$
   (c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (d) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

4. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is **[2010]**

   (a) $x + 2y - 2z = 0$ (b) $3x + 2y - 2z = 0$
   (c) $x - 2y + z = 0$ (d) $5x + 2y - 4z = 0$

5. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals **[2009]**

   (a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

6. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$

   Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is **[2009]**

   (a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

7. A plane which is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, passes through (1, –2, 1). The distance of the plane from the point (1, 2, 2) is **[2006 - 3M, –1]**

   (a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

8. A variable plane at a distance of the one unit from the origin cuts the coordinates axes at A, B and C. If the centroid $D(x, y, z)$ of triangle ABC satisfies the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then the value k is **[2005S]**

   (a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 9

9. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is **[2003S]**

   (a) 7 (b) –7
   (c) no real value (d) 4

2 Integer Value Answer/ Non-Negative Integer

10. Let $\overrightarrow{OP} = \frac{\alpha - 1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \frac{\beta - 1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $a, b \in \mathbb{R} - \{0\}$ and O denotes the origin. If $(\overrightarrow{OP} \times \overrightarrow{OQ}).\overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$ then the value of l is ______. **[Adv. 2024]**

11. Let P be the plane $\sqrt{3}x+2y+3z=16$ and let $S=\left\{\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}:\alpha^2+\beta^2+\gamma^2=1 \text{ and the distance of } (\alpha,\beta,\gamma) \text{ from the plane P is } \frac{7}{2}\right\}$. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is **[Adv. 2023]**

12. Let P be a point in the first octant, whose image Q in the plane $x+y=3$ (that is, the line segment PQ is perpendicular to the plane $x+y=3$ and the mid-point of PQ lies in the plane $x+y=3$) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____. **[Adv. 2018]**

13. If the distance between the plane $Ax-2y+z=d$ and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then find $|d|$. **[2010]**

3 Numeric/ New Stem Based Questions

14. Three lines are given by $\vec{r}=\lambda\hat{i},\lambda\in R$; $\vec{r}=\mu(\hat{i}+\hat{j}),\mu\in R$ and $\vec{r}=v(\hat{i}+\hat{j}+\hat{k}),v\in R$. Let the lines cut the plane $x+y+z=1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals _____. **[Adv. 2019]**

4 Fill in the Blanks

15. A nonzero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is .... **[1996 - 2 Marks]**

16. The unit vector perpendicular to the plane determined by $P(1,-1,2)$, $Q(2,0,-1)$ and $R(0,2,1)$ is .... **[1983 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

17. Let $\mathbb{R}$ denote the three-dimensional space. Take two points $P=(1,2,3)$ and $Q=(4,2,7)$. Let dist (X, Y) denote the distance between two points X and Y in $\mathbb{R}^3$. Let $S=\{X\in\mathbb{R}^3:(\text{dist}(X,P))^2-(\text{dist}(X,Q))^2=50\}$ and $T=\{Y\in\mathbb{R}^3:(\text{dist}(Y,Q))^2-(\text{dist}(Y,P))^2=50\}$. Then which of the following statements is (are) TRUE? **[Adv. 2024]**

(a) There is a triangle whose area is 1 and all of whose vertices are from S.

(b) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(c) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(d) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

18. A straight line drawn from the point P (1, 3, 2), parallel to the line $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$, intersects the plane $L_1: x-y+3z=6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2: 2x-y+z=-4$ at the point R. Then which of the following statements is (are) TRUE? **[Adv. 2024]**

(a) The length of the line segment PQ is $\sqrt{6}$

(b) The coordinates of R are (1, 6, 3)

(c) The centroid of the triangle PQR is $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$

(d) The perimeter of the triangle PQR is $\sqrt{2}+\sqrt{6}+\sqrt{11}$

19. Let P_1 and P_2 be two planes given by **[Adv. 2022]**

$P_1: 10x+15y+12z-60=0$,

$P_2: -2x+5y+4z-20=0$.

Which of the following straight lines can be an edge of some tetrahedron whose two faces lie on P_1 and P_2?

(a) $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}$ (b) $\frac{x-6}{-5}=\frac{y}{2}=\frac{z}{3}$

(c) $\frac{x}{-2}=\frac{y-4}{5}=\frac{z}{4}$ (d) $\frac{x}{1}=\frac{y-4}{-2}=\frac{z}{3}$

20. Let S be the reflection of a point Q with respect to the plane given by $\vec{r}=-(t+p)\hat{i}+t\hat{j}+(1+p)\hat{k}$ where t, p are real parameters and $\hat{i},\hat{j},\hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of Q and S are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are TRUE? **[Adv. 2022]**

(a) $3(\alpha+\beta)=-101$ (b) $3(\beta+\gamma)=-71$

(c) $3(\gamma+\alpha)=-86$ (d) $3(\alpha+\beta+\gamma)=-121$

21. Let L_1 and L_2 be the following straight line.

$L_1:\frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3}$ and $L_2:\frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}$.

Suppose the straight line $L:\frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$ lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE? **[Adv. 2020]**

(a) $\alpha-\gamma=3$ (b) $l+m=2$

(c) $\alpha-\gamma=1$ (d) $l+m=0$

22. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2+\beta^2+\gamma^2 \neq 0$ and $\alpha+\gamma=1$. Suppose the point (3, 2, –1) is the mirror image of the point (1, 0, –1) with respect to the plane $\alpha x+\beta y+\gamma z=\delta$.
Then which of the following statements is/are TRUE?
(a) $\alpha+\beta=2$ (b) $\delta-\gamma=3$ [Adv. 2020]
(c) $\delta+\beta=4$ (d) $\delta+\beta+\gamma=\delta$

23. Let $P_1: 2x+y-z=3$ and $P_2: x+2y+z=2$ be two planes. Then, which of the following statement(s) is (are) TRUE? [Adv. 2018]
(a) The line of intersection of P_1 and P_2 has direction ratios 1, 2, – 1
(b) The line $\frac{3x-4}{9}=\frac{1-3y}{9}=\frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
(c) The acute angle between P_1 and P_2 is 60°.
(d) If P_3 is the plane passing through the point (4, 2, – 2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

24. Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, and OP and OR along the x–axis and the y–axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point, T of diagonal OQ such that TS = 3. Then [Adv. 2016]
(a) the acute angle between OQ and OS is $\frac{\pi}{3}$
(b) the equation of the plane containing the triangle OQS is $x - y = 0$
(c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

25. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M? [Adv. 2015]
(a) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$
(c) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ (d) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

26. In R^3, consider the planes $P_1: y=0$ and $P_2: x+z=1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true? [Adv. 2015]
(a) $2\alpha+\beta+2\gamma+2=0$ (b) $2\alpha-\beta+2\gamma+4=0$
(c) $2\alpha+\beta-2\gamma-10=0$ (d) $2\alpha-\beta+2\gamma-8=0$

27. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) [Adv. 2013]
(a) 1 (b) 2 (c) 3 (d) 4

28. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane (s) containing these two lines is (are) [2012]
(a) $y+2z=-1$ (b) $y+z=-1$
(c) $y-z=-1$ (d) $y-2z=-1$

29. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and that P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and a given vector $2\hat{i}+\hat{j}-2\hat{k}$ is [2006 - 5M, –1]
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

7 Match the Following

30. Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1=\lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2=(\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X.
Match each entry in **List-I** to the correct entries in **List-II**.

List-I	List-II
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$

(R) The distance of origin from H_0 is (3) 0

(S) The distance of origin from the point of intersection of planes $y = z$, $x = 1$ and H_0 is (4) $\sqrt{2}$

(5) $\frac{1}{\sqrt{2}}$

The correct option is: **[Adv. 2023]**

(a) (P) → (2), (Q) → (4), (R) → (5), (S) → (1)
(b) (P) → (5), (Q) → (4), (R) → (3), (S) → (1)
(c) (P) → (2), (Q) → (1), (R) → (3), (S) → (2)
(d) (P) → (5), (Q) → (1), (R) → (4), (S) → (2)

31. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 = 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists : **[Adv. 2013]**

List I	List II
P. $a =$	1. 13
Q. $b =$	2. −3
R. $c =$	3. 1
S. $d =$	4. −2

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

32. Match the statements/expressions given in **Column-I** with the values given in **Column-II.** **[2009]**

Column-I	Column-II
(A) The number of solutions of the equation $x\,e^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution(s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$, then value(s) of $y(\ln 2)$	(s) 4
	(t) 5

33. Consider the following linear equations

$ax + by + cz = 0; \quad bx + cy + az = 0; \quad cx + ay + bz = 0$

Match the conditions/expressions in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the *ORS*. **[2007]**

Column I	Column II
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at asingle point
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

34. Match the following : [2006 - 6M]

Column I	Column II
(A) Two rays $x+y=\|a\|$ and $ax-y=1$ intersects each other in the first quadrant in the interval $a\in(a_0,\infty)$, the value of a_0 is	(p) 2
(B) Point (α,β,γ) lies on the plane $x+y+z=2$. Let $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\hat{k}\times(\hat{k}\times\vec{a})=0$, then $\gamma=$	(q) $\frac{4}{3}$
(C) $\left\|\int_0^1(1-y^2)\,dy\right\|+\left\|\int_1^0(y^2-1)\,dy\right\|$	(r) $\left\|\int_0^1\sqrt{1-x}\,dx\right\|+\left\|\int_{-1}^0\sqrt{1+x}\,dx\right\|$
(D) If $\sin A\sin B\sin C+\cos A\cos B=1$, then the value of $\sin C=$	(s) 1

8 Comprehension/Passage Based Questions

Consider the lines

$$L_1:\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}\qquad L_2:\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$$

[2008]

35. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

36. The shortest distance between L_1 and L_2 is

(a) 0 (b) $\frac{17}{\sqrt{3}}$ (c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$

37. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(a) $\frac{2}{\sqrt{75}}$ (b) $\frac{7}{\sqrt{75}}$ (c) $\frac{13}{\sqrt{75}}$ (d) $\frac{23}{\sqrt{75}}$

9 Assertion and Reason/Statement Type Questions

38. Consider three planes

$P_1: x-y+z=1$ $P_2: x+y-z=-1$

$P_3: x-3y+3z=2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

STATEMENT - 1 : At least two of the lines L_1, L_2 and L_3 are non-parallel and

STATEMENT - 2 : The three planes doe not have a common point. [2008]

(A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

(B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

(C) STATEMENT - 1 is True, STATEMENT - 2 is False

(D) STATEMENT - 1 is False, STATEMENT - 2 is True

39. Consider the planes $3x-6y-2z=15$ and $2x+y-2z=5$.

STATEMENT-1 : The parametric equations of the line of intersection of the given planes are $x=3+14t, y=1+2t, z=15t$. **because**

STATEMENT-2 : The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of given planes. [2007 -3 marks]

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

10 Subjective Problems

40. Find the equation of the plane containing the line $2x-y+z-3=0, 3x+y+z=5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1). [2005 - 2 Marks]

41. P_1 and P_2 are planes passing through origin. L_1 and L_2 are two line on P_1 and P_2 respectively such that their intersection is origin. Show that there exists points A, B, C, whose permutation A', B', C' can be chosen such that (i) A is on L_1, B on P_1 but not on L_1 and C not on P_1 (ii) A' is on L_2, B' on P_2 but not on L_2 and C' not on P_2. [2004 - 4 Marks]

42. A parallelepiped 'S' has base points A, B, C and D and upper face points A', B', C' and D'. This parallelepiped is compressed by upper face $A'B'C'D'$ to form a new parallelepiped 'T' having upper face points A'', B'', C'' and D''. Volume of parallelepiped T is 90 percent of the volume of parallelepiped S. Prove that the locus of 'A''', is a plane. [2004 - 2 Marks]

43. Find the equation of plane passing through (1, 1, 1) & parallel to the lines L_1, L_2 having direction ratios (1,0,–1), (1,–1,0). Find the volume of tetrahedron formed by origin and the points where these planes intersect the coordinate axes. [2004 - 2 Marks]

44. (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. [2003 - 4 Marks]

Answer Key

Topic-1 : Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1. (b) **2.** (c)

Topic-2 : Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1. (a) **2.** (b) **3.** (a,d) **4.** (a,b,d) **5.** (c) **6.** (b,d) **7.** (c) **8.** (A)-t; (B)-p, r; (C)-q, s; (D)-r

Topic-3 : Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1. (a) **2.** (a) **3.** (a) **4.** (c) **5.** (c) **6.** (a) **7.** (d) **8.** (d) **9.** (a) **10.** (5)

11. (45) **12.** (8) **13.** (6) **14.** (0.75) **15.** $\frac{\pi}{4}, \frac{3\pi}{4}$ **16.** $\pm\frac{(2\hat{i}+\hat{j}+\hat{k})}{\sqrt{6}}$ **17.** (a,b, c) **18.** (a,c) **19.** (a,b)

20. (a,b,c) **21.** (a,b) **22.** (a,b,c) **23.** (c,d) **24.** (b,c,d) **25.** (a,b) **26.** (b,d) **27.** (a,d) **28.** (b,c) **29.** (b,d)

30. (b) **31.** (a) **32.** (A)-p; (B)-q, s; (C)-q, r, s, t; (D)-t **33.** (A)-r; (B)-q; (C)-p; (D)-s

34. (A)-s; (B)-p; (C)-q, r; (D)-s **35.** (b) **36.** (d) **37.** (c) **38.** (d) **39.** (d)

Hints & Solutions

Topic-1: Direction Ratios & Direction cosines of a Line, Angle between two lines in terms of dc's and dr's, Projection of a Point on a Line

1. **(b)** As per question, direction cosines of the line :

$\ell = \cos 45° = \frac{1}{\sqrt{2}}$, $m = \cos 120° = \frac{-1}{2}$, $n = \cos\theta$

where θ is the angle, which line makes with positive z-axis.

We know that, $\ell^2 + m^2 + n^2 = 1$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \quad (\theta \text{ being acute})$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

2. **(c)** As per question the direction cosines of the line are $\cos\theta$, $\cos\beta$, $\cos\theta$

$\therefore \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\because 2\cos^2\theta = 1 - \cos^2\theta$

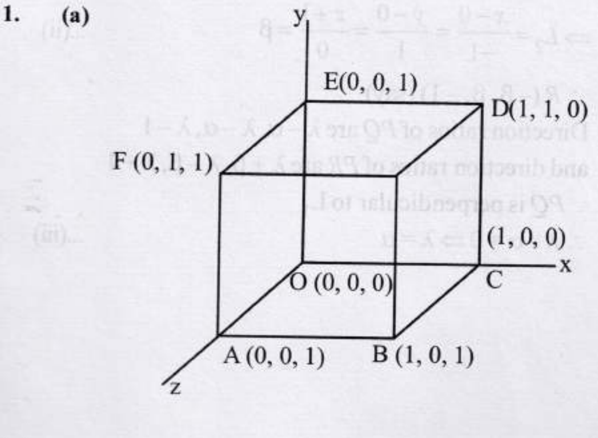
$\Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)

$\Rightarrow 2\cos^2\theta = 3 - 3\cos^2\theta$

$\therefore \cos^2\theta = \frac{3}{5}$

Topic-2: Equation of a Straight Line in Cartesian and Vector Form, Angle Between two Lines, Distance Between two Parallel Lines

1. **(a)**



Equation of face diagonal OD line is

$l_1 : \vec{r} = \lambda(\hat{i} + \hat{j})$

Equation of main diagonal BE is

$l_2 : \vec{r} = \hat{j} + \mu(\hat{i} - \hat{j} + \hat{k})$

$$\text{Shortest distance} = \left|\frac{\hat{j}.(\hat{i}+\hat{j})\times(\hat{i}-\hat{j}+\hat{k})}{(\hat{i}+\hat{j})\times(\hat{i}-\hat{j}+\hat{k})}\right|$$

$$= \left|\frac{\hat{j}.(\hat{i}-\hat{j}-2\hat{k})}{\hat{i}-\hat{j}-2\hat{k}}\right| = \frac{1}{\sqrt{6}}$$

In other case S.D is zero.

2. **(b)** Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

$\Rightarrow x = 2\lambda + 1, y = 3\lambda - 1$ and $z = 4\lambda + 1$

and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$

$\Rightarrow x = 3 + \mu, y = k + 2\mu$ and $z = \mu$

Since given lines intersect each other

$\Rightarrow 2\lambda + 1 = 3 + \mu$ ...(i)

$3\lambda - 1 = 2\mu + k$ ...(ii)

$\mu = 4\lambda + 1$ ...(iii)

Solving (i) and (iii) and putting the value of λ and μ in (ii) we get, $k = \frac{9}{2}$

3. **(a, d)** Let any point

$P(\lambda, 0, 0)$ on L_1, $Q(0, \mu, 1)$ on L_2 and $R(1, 1, \nu)$ on L_3

$\because$ P, Q, R are collinear, $\therefore \overline{PQ} \parallel \overline{PR}$

$$\Rightarrow \frac{\lambda}{\lambda - 1} = \frac{-\mu}{-1} = \frac{-1}{-\nu}$$

$$\Rightarrow \mu = \frac{\lambda}{\lambda - 1}, \nu = \frac{\lambda - 1}{\lambda}$$

Clearly from above that $\lambda \neq 0, 1$

$$\therefore Q\left(0, \frac{\lambda}{\lambda - 1}, 1\right)$$

(a) For $Q = \hat{k} - \frac{1}{2}\hat{j}$

$$\frac{\lambda}{\lambda-1}=-\frac{1}{2}\Rightarrow 3\lambda=+1\text{, which is possible.}$$

(b) For $Q=\hat{k}$

$$\frac{\lambda}{\lambda-1}=0\Rightarrow\lambda=0\text{, not possible}$$

(c) For $Q=\hat{k}+\hat{j}$

$$\frac{\lambda}{\lambda-1}=1\Rightarrow\lambda=\lambda-1,\ \text{not possible}$$

(d) For $Q=\hat{k}+\frac{1}{2}\hat{j}$

$$\frac{\lambda}{\lambda-1}=\frac{1}{2}\Rightarrow 2\lambda=\lambda-1\Rightarrow\lambda=-1,$$

which is possible

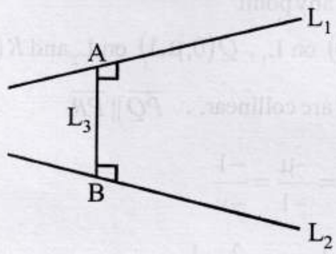
Hence options (a) and (d) are correct and options (b) and (c) are incorrect.

4. **(a, b, d)** $L_1:\vec{r}=\hat{i}+\lambda(-i+2j+2\hat{k})$ $L_2:\vec{r}$ $=\mu(2i-j+2\hat{k})$

Since L_3 being perpendicular to both L_1 and L_2, is the shortest distance line between L_1 & L_2.

$\therefore$ Direction vector of line L_3 : $(-\hat{i}+2\hat{j}+2\hat{k})\times(2\hat{i}-\hat{j}+2\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 2\\ 2 & -1 & 2\end{vmatrix}=6\hat{i}+6\hat{j}-3\hat{k}$$



$\because$ L_1 and L_2 are skew lines

Let any point on L_1 and L_2 be

$A(1-\lambda, 2\lambda, 2\lambda)$ and $B(2\mu, -\mu, 2\mu)$.

$\therefore$ dr's of AB $=2\mu+\lambda-1, -\mu-2\lambda, 2\mu-2\lambda$

$\because$ AB and L_3 are representing the same line

$$\therefore\quad \frac{2\mu+\lambda-1}{6}=\frac{-\mu-2\lambda}{6}=\frac{2\mu-2\lambda}{-3}$$

$$\Rightarrow\quad 3\lambda+3\mu=1 \qquad\text{...(i)}$$

$$6\lambda-3\mu=0 \qquad\text{...(ii)}$$

Solving (i) and (ii) we get : $\lambda=\frac{1}{9},\mu=\frac{2}{9}$

$$\therefore\quad A\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right)\text{ and }B\left(\frac{4}{9},\frac{-2}{9},\frac{4}{9}\right)$$

$\therefore$ Equation of L_3 is given by

$$\vec{r}=\frac{2}{9}(4i+\hat{j}+\hat{k})+t(2i+2j-\hat{k})$$

$\therefore$ (a) is correct.

or $\vec{r}=\frac{2}{9}(2\hat{i}-\hat{j}+2\hat{k})+t(2i+2j-\hat{k})$

$\therefore$ (b) is correct

Also mid-point of AB is $\left(\frac{2}{3},0,\frac{1}{3}\right)$

$\therefore$ L_3 can also be written as

$$\vec{r}=\frac{1}{3}(2\hat{i}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k})\text{, where } t\in R$$

$\therefore$ (d) is correct.

Clearly (0, 0, 0) does not lie on

$$\vec{r}=\frac{2}{9}(4\hat{i}+\hat{j}+\hat{k})+t(2\hat{i}+2\hat{j}-\hat{k})$$

$\therefore$ $\vec{r}=t(2\hat{i}+2\hat{j}-\hat{k})$ can not describe the line L_3.

$\therefore$ (c) is incorrect.

5. **(c)** Given that lines are $x=y, z=1$

$$\Rightarrow L_1=\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-1}{0}=\alpha \qquad\text{...(i)}$$

$\therefore Q(\alpha,\alpha,1)$

and $y=-x, z=-1$

$$\Rightarrow L_2=\frac{x-0}{-1}=\frac{y-0}{1}=\frac{z+1}{0}=\beta \qquad\text{...(ii)}$$

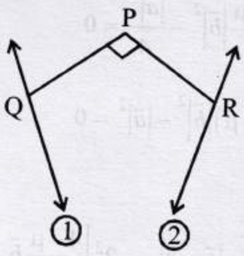
$\therefore R(-\beta,\beta,-1)$ (say)

Direction ratios of PQ are $\lambda-\alpha, \lambda-\alpha, \lambda-1$

and direction ratios of PR are $\lambda+\beta, \lambda-\beta, \lambda+1$

$\because PQ$ is perpendicular to L_1

$\therefore \lambda-\alpha=0\Rightarrow\lambda=\alpha \qquad\text{...(iii)}$



$\because$ PR is perpendicular to L_2

$\therefore -(\lambda+\beta)+\lambda-\beta=0 \Rightarrow \beta=0$

$\therefore$ dr's of PQ are $0, 0, \lambda-1$

and dr's of PR are $\lambda, \lambda, \lambda+1$

$\because \angle QPR = 90° \Rightarrow (\lambda-1)(\lambda+1)=0 \Rightarrow \lambda=1$ or -1

But for $\lambda=1$, we get point Q itself

$\therefore$ we take $\lambda=-1$

6. **(b, d)** The given lines are

$$\ell_1 : \left(3\hat{i}-\hat{j}+4\hat{k}\right)+t\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

$$\ell_2 : \left(3\hat{i}+3\hat{j}+2\hat{k}\right)+s\left(2\hat{i}+2\hat{j}+\hat{k}\right)$$

Direction vector perpendicular to bo ℓ_1 and ℓ_2

$$\vec{b}=\ell_1\times\ell_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 2 & 2 & 1\end{vmatrix}=-2\hat{i}+3\hat{j}-2\hat{k}$$

$$\therefore \quad \ell : \frac{x}{2}=\frac{y}{-3}=\frac{z}{2}=\lambda$$

Any point on ℓ_1 is $(t+3, 2t-1, 2t+4)$ and any point on ℓ is $(2\lambda, -3\lambda, 2\lambda)$

$\therefore$ Let intersection point of ℓ and ℓ_1 be P.

$t+3=2\lambda,\ 2t-1=-3\lambda,\ 2t+4=2\lambda$

$\Rightarrow \quad t=-1,\ \lambda=1 \qquad \therefore P(2,-3,2)$

Any point Q on ℓ_2 is $(2s+3, 2s+3, s+2)$

According to question $PQ=\sqrt{17}$

$\Rightarrow \quad (2s+1)^2+(2s+6)^2+s^2=17$

$\Rightarrow \quad 9s^2+28s+20=0 \Rightarrow s=-2, \dfrac{-10}{9}$

$\therefore$ Point Q can be $(-1,-1,0)$ and $\left(\dfrac{7}{9},\dfrac{7}{9},\dfrac{8}{9}\right)$

7. (c) Let $L_1 : \dfrac{x+11}{1}=\dfrac{y+21}{2}=\dfrac{z+29}{3}=\lambda$

and $L_2 : \dfrac{x+16}{3}=\dfrac{y+11}{2}=\dfrac{z+4}{\gamma}=\mu$

$x=\lambda-11=3\mu-16 \Rightarrow \lambda-3\mu=-5$ ...(i)

$y=2\lambda-21=2\mu-11 \Rightarrow 2\lambda-2\mu=10$ ...(ii)

$z=3\lambda-29=\mu\gamma-4 \Rightarrow 3\lambda-\mu\gamma=25$ ...(iii)

from (i) & (ii)

$\lambda=10, \mu=5$

Now from (iii)

$3(10)-5\gamma=25 \qquad \therefore \gamma=1$

So, $R_1 \equiv (-1,-1,1)$

Now, $OR_1=-\hat{i}-\hat{j}+\hat{k}$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 3 & 2 & 1\end{vmatrix}=-4\hat{i}-(-8)\hat{j}-4\hat{k}$$

$$\vec{n}=-4\hat{i}+8\hat{j}+4\hat{k}=-4(\hat{i}-2\hat{j}+\hat{k})$$

$$\hat{n}=\pm\frac{4(\hat{i}-2\hat{j}+\hat{k})}{4\sqrt{6}}=\pm\frac{(\hat{i}-2\hat{j}+\hat{k})}{\sqrt{6}}$$

$$\overline{OR}.\hat{n}=\pm(-\hat{i}-\hat{j}+\hat{k})\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

$$=\pm\frac{2}{\sqrt{6}}=\pm\sqrt{\frac{4}{6}}=\pm\sqrt{\frac{2}{3}}$$

8. **(A) → t ; (B) → p,r ; (C) → q,s ; (D) → r**

Let the line through origin be $L : \dfrac{x}{a}=\dfrac{y}{b}=\dfrac{z}{c}$ ....(i)

since line L intersects

$L_1 : \dfrac{x-2}{1}=\dfrac{y-1}{-2}=\dfrac{z+1}{1}$ .....(ii)

and $L_2 : \dfrac{x-8/3}{2}=\dfrac{y+3}{-1}=\dfrac{z-1}{1}$ ....(iii)

at P and Q,

$\therefore$ line L and L_1 coplaner.

$$\therefore \text{Using} \begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$$

we get $\begin{vmatrix}2 & 1 & -1\\ a & b & c\\ 1 & -2 & 1\end{vmatrix}=0 \Rightarrow a+3b+5c=0$ ...(iv)

Also L and L_2 coplaner

and $\begin{vmatrix} 8/3 & -3 & 1 \\ a & b & c \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow 3a+b-5c=0$ ...(v)

On solving (iv) and (v), we get

$$\frac{a}{-15-5}=\frac{b}{15+5}=\frac{c}{1-9} \text{ or } \frac{a}{5}=\frac{b}{-5}=\frac{c}{2}$$

Hence equation (i) becomes $\frac{x}{5}=\frac{y}{-5}=\frac{z}{2}=\lambda$

Any point on L, $P(5\lambda,-5\lambda,2\lambda)$

which lies on (ii) also

$$\therefore \frac{5\lambda-2}{1}=\frac{-5\lambda-1}{-2}=\frac{2\lambda+1}{1} \Rightarrow \lambda=1$$

$\therefore$ P (5,–5,2)

Also Any point on L, $Q(5\lambda,-5\lambda,2\lambda)$

which lies on (iii) also

$$\therefore \frac{5\lambda-8/3}{3}=\frac{-5\lambda+3}{-1}=\frac{2\lambda-1}{1} \Rightarrow \lambda=2/3$$

$$\therefore Q\left(\frac{10}{3},\frac{-10}{3},\frac{4}{3}\right)$$

Hence $d^2 = PQ^2 = \left(\frac{25}{9}+\frac{25}{9}+\frac{4}{9}\right)=6$

(B) $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\tan^{-1}\frac{3}{4}$

$$\left[\because \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{x+3-x+3}{1+x^2-9}\right)=\tan^{-1}\left(\frac{3}{4}\right), x^2-9>-1$$

$$\Rightarrow \frac{6}{x^2-8}=\frac{3}{4} \Rightarrow x^2=16 \text{ or } x=4,-4$$

(C) $\because \vec{a}=\mu\vec{b}+4\vec{c} \Rightarrow \vec{c}=\frac{\vec{a}-\mu\vec{b}}{4}$

Then $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$

$$\Rightarrow (\vec{b}-\vec{a}).\left(\vec{b}+\frac{\vec{a}-\mu\vec{b}}{4}\right)=0$$

$$\Rightarrow (\vec{b}-\vec{a}).\left(\frac{4-\mu}{4}\vec{b}+\frac{\vec{a}}{4}\right)=0$$

$$\Rightarrow \frac{4-\mu}{4}|\vec{b}|^2-\frac{|\vec{a}|^2}{4}=0$$

$$\Rightarrow (4-\mu)|\vec{b}|^2-|\vec{a}|^2=0 \quad ...(i)$$

Also,

$$2|\vec{b}+\vec{c}|=|\vec{b}-\vec{a}| \Rightarrow 2^2\left|\frac{4-\mu}{4}\vec{b}+\frac{\vec{a}}{4}\right|^2=|\vec{b}-\vec{a}|^2$$

$$\Rightarrow (4-\mu)^2|\vec{b}|^2+|\vec{a}|^2=4|\vec{b}|^2+4|\vec{a}|^2 \left[\because \vec{a}.\vec{b}=0\right]$$

$$\Rightarrow \left[(4-\mu)^2-4\right]|\vec{b}|^2=3|\vec{a}|^2 \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{(4-\mu)^2-4}{4-\mu}=\frac{3}{1}$$

$$\Rightarrow \mu^2-8\mu+12=12-3\mu \Rightarrow \mu^2-5\mu=0$$

$$\Rightarrow \mu=0 \text{ or } 5$$

(D) $I=\frac{2}{\pi}\int_{-\pi}^{\pi}\frac{\sin(9x/2)}{\sin(x/2)}dx=\frac{2}{\pi}\times 2\int_0^{\pi}\frac{\sin 9x/2}{\sin x/2}dx$

[$\because f(x)$ is even function]

Let $\frac{x}{2}=\theta \Rightarrow dx=2d\theta$

Also at $x=0,\theta=0$ and at $x=\pi,\theta=\pi/2$

$$\therefore I=\frac{8}{\pi}\int_0^{\pi/2}\frac{\sin 9\theta}{\sin\theta}d\theta$$

$$I=\frac{8}{\pi}\int_0^{\pi/2}\left[\frac{\sin 9\theta-\sin 7\theta}{\sin\theta}+\frac{(\sin 7\theta-\sin 5\theta)}{\sin\theta}+\frac{(\sin 5\theta-\sin 3\theta)}{\sin\theta}+\frac{(\sin 3\theta-\sin\theta)}{\sin\theta}+\frac{\sin\theta}{\sin\theta}\right]d\theta$$

$$=\frac{16}{\pi}\int_0^{\pi/2}(\cos 8\theta+\cos 6\theta+\cos 4\theta+\cos 2\theta)d\theta+\frac{8}{\pi}\int_0^{\pi/2}d\theta$$

$$=\frac{16}{\pi}\left[\frac{\sin 8\theta}{8}+\frac{\sin 6\theta}{6}+\frac{\sin 4\theta}{4}+\frac{\sin 2\theta}{2}\right]_0^{\pi/2}+\frac{8}{\pi}(\theta)_0^{\pi/2}$$

$$=0+\frac{8}{\pi}\left(\frac{\pi}{2}-0\right)=4$$

Topic-3: Equation of a Plane in Different Forms, Equation of a Plane Passing Through the Intersection of two Given Planes, Projection of a Line on a Plane

1. **(a)** Equation of the plane passing through the intersection line of given planes is

$(x+2y+3z-2)+\lambda(x-y+z-3)=0$

or $(1+\lambda)x+(2-\lambda)y+(3+\lambda)z+(-2-3\lambda)=0$

$\because$ Its distance from the point $(3, 1, -1)$ is $\frac{2}{\sqrt{3}}$

$$\therefore \left|\frac{3(1+\lambda)+1(2-\lambda)-1(3+\lambda)+(-2-3\lambda)}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}\right|=\frac{2}{\sqrt{3}}$$

$$\Rightarrow \left|\frac{-2\lambda}{\sqrt{3\lambda^2+4\lambda+14}}\right|=\frac{2}{\sqrt{3}}$$

$$\Rightarrow 3\lambda^2+4\lambda+14=3\lambda^2 \Rightarrow \lambda=-\frac{7}{2}$$

$\therefore$ Required equation of plane is

$$(x+2y+3z-2)-\frac{7}{2}(x-y+z-3)=0$$

or $5x-11y+z=17$

2. **(a)** Equation of st. line joining $Q(2, 3, 5)$ and $R(1, -1, 4)$ is

$$\frac{x-2}{-1}=\frac{y-3}{-4}=\frac{z-5}{1}=\lambda$$

Let $P(-\lambda+2, -4\lambda+3, -\lambda+5)$

Since P also lies on $5x-4y-z=1$

$\therefore -5\lambda+10+16\lambda-12+\lambda-5=1$

$$\Rightarrow 12\lambda=8 \Rightarrow \lambda=\frac{2}{3} \quad \therefore P=\left(\frac{4}{3},\frac{1}{3},\frac{13}{3}\right)$$

Now let another point S on QR be

$(-\mu+2, -4\mu+3, -\mu+5)$

Since S is the foot of perpendicular drawn from $T(2, 1, 4)$ to QR, where dr's of ST are $\mu, 4\mu-2, \mu-1$ and dr's of QR are $-1, -4, -1$

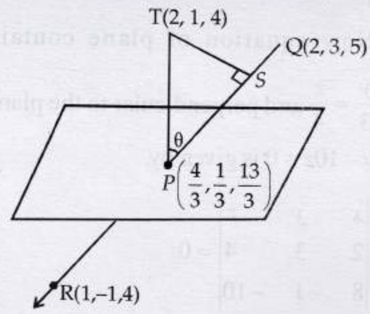
$$\therefore \quad -\mu-16\mu+8-\mu+1=0 \Rightarrow 18\mu=9 \Rightarrow \mu=\frac{1}{2}$$

$$\therefore \quad S=\left(\frac{3}{2},1,\frac{9}{2}\right)$$

$\therefore$ Distance between P and S

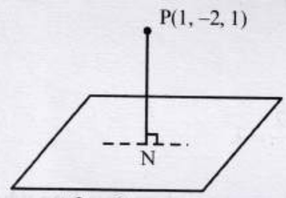
$$=\sqrt{\left(\frac{4}{3}-\frac{3}{2}\right)^2+\left(\frac{1}{3}-1\right)^2+\left(\frac{13}{3}-\frac{9}{2}\right)^2}$$

$$=\sqrt{\frac{1}{36}+\frac{4}{9}+\frac{1}{36}}=\frac{1}{\sqrt{2}}$$



3. **(a)** Since perpendicular distance of $x+2y-2z-\alpha=0$ from the point $(1, -2, 1)$ is 5

$$\therefore \left|\frac{1-4-2-\alpha}{3}\right|=5$$



$$\Rightarrow \frac{-5-\alpha}{3}=5 \text{ or } -5$$

$\Rightarrow \alpha=-20$ or 10

But $\alpha>0 \Rightarrow \alpha=10$

$\therefore$ Equation of plane : $x+2y-2z-10=0$

We know that foot of perpendicular from point (x, y, z) to the plane $ax+by+cz+d=0$ is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-(ax_1+by_1+cz_1+d)}{(a^2+b^2+c^2)}$$

$$\therefore \quad \frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=\frac{-(1-4-2-10)}{9}=\frac{5}{3}$$

$$\Rightarrow \quad x=\frac{8}{3}, y=\frac{4}{3}, z=-\frac{7}{3}$$

$$\therefore \quad \text{Foot of } \perp^r \equiv \left(\frac{8}{3},\frac{4}{3},\frac{-7}{3}\right)$$

4. (c) Equation of plane containing two lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is given by

$$\begin{vmatrix} x & y & z \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix}=0 \Rightarrow 8x-y-10z=0$$

Now equation of plane containing the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane $8x-y-10z=0$ is given by,

$$\begin{vmatrix} x & y & z \\ 2 & 3 & 4 \\ 8 & -1 & -10 \end{vmatrix}=0$$

$$\Rightarrow -26x+52y-26z=0 \quad \text{or} \quad x-2y+z=0$$

5. (c) Since line makes equal angle with coordinate axes and which has positive direction cosines

$\therefore$ D-c's $=\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

$\Rightarrow$ D-r's = 1, 1, 1

$\therefore$ Equation of line is

$$\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-2}{1}=\lambda$$

$\therefore Q(\lambda+2,\lambda-1,\lambda+2)$ be any point on this line where it meets the plane $2x+y+z=9$

$\Rightarrow 2(\lambda+2)+\lambda-1+\lambda+2=9 \Rightarrow \lambda=1$

$\therefore$ Q has coordintes (3, 0, 3)

$\therefore$ $PQ=\sqrt{(2-3)^2+(-1-0)^2+(2-3)^2}=\sqrt{3}$

6. (a) $\because \vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$

$=(1-3\mu)\hat{i}+(-1+\mu)\hat{j}+(2+5\mu)\hat{k}$

Let coordinates of Q be $(-3\mu+1,\mu-1,5\mu+2)$

$\therefore$ d.r's of $\overrightarrow{PQ}=-3\mu-2,\ \mu-3,\ 5\mu-4$

Given that $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$

$\therefore$ $1.(-3\mu-2)-4.(\mu-3)+3.(5\mu-4)=0$

$\Rightarrow 8\mu=2$ or $\mu=\frac{1}{4}$

7. (d) We know that the equation of plane through the point (1, –2, 1) and perpendicular to the planes $2x-2y+z=0$ and $x-y+2z=4$ is

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}=0 \Rightarrow x+y+1=0$$

It's distance from the point (1, 2, 2) is

$$\left|\frac{1+2+1}{\sqrt{2}}\right|=2\sqrt{2}.$$

8. (d) Let $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ be the eq^n of variable plane which meets the axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0.0, c)$.

$\therefore$ Centroid of ΔABC is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

$\Rightarrow x=\frac{a}{3}, y=\frac{b}{3}, z=\frac{c}{3}$

putting these values in

$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k \Rightarrow \frac{9}{a^2}+\frac{9}{b^2}+\frac{9}{c^2}=k$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{z^2}=\frac{k}{9}$ ...(i)

Also given that the distance of plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ from (0, 0, 0) is 1 unit.

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1 \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1$$

From (i) we get $\frac{k}{9}=1$ i.e. $k=9$

9. (a) Since the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, then the point $(4, 2, k)$ on line also lie on the given plane and hence

$2\times4-4\times2+k=7 \Rightarrow k=7$

10. (5) Given that $(\overrightarrow{OP}\times\overrightarrow{OQ}).\overrightarrow{OR}=0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix}=0$$

$$\Rightarrow \frac{\alpha-1}{\alpha}\left(\frac{\beta-1}{2\beta}-1\right)-\left(\frac{1}{2}-1\right)+1\left(1-\frac{\beta-1}{\beta}\right)=0$$

$$\frac{\alpha-1}{\alpha}\left(\frac{-\beta-1}{2\beta}\right)+\frac{1}{2}+\frac{1}{\beta}=0$$

$$\Rightarrow \frac{\beta+2}{2\beta}$$

$$=\frac{\alpha\beta+\alpha-\beta-1}{2\alpha\beta}$$

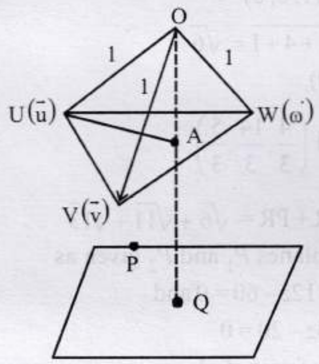
$$\Rightarrow \alpha\beta+2\alpha=\alpha\beta+\alpha-\beta-1 \Rightarrow \alpha+\beta+1=0 \quad ...(i)$$

Since, $(\alpha, \beta, 2)$ lies on plane $3x+3y-z+l=0$

$\Rightarrow 3(\alpha+\beta)-2+l=0$ ...(ii)

$\Rightarrow -3-2+l=0 \Rightarrow l=5$

11. **(45)**



Given, $|\vec{u}-\vec{v}|=|\vec{v}-\vec{w}|=|\vec{w}-\vec{u}|$

So, ΔUVW is one equilateral triangle

Given that distances of points U, V, W from plane $P=\frac{7}{2} \Rightarrow AQ=\frac{7}{2}$

Distance of plane P from origin

$$=\left|\frac{0+0+0-16}{\sqrt{3+4+9}}\right|=4=OQ$$

$$\therefore OA=OQ-AQ=4-\frac{7}{2}=\frac{1}{2}$$

In ΔOAU, $UA=\sqrt{OV^2-OA^2}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}=R$

In ΔUVW, is circumcenter

$US=R\cos 30^\circ \Rightarrow UV=2R\cos 30^\circ=\frac{3}{2}$

$$\therefore \text{Ar } \Delta UVW=\frac{\sqrt{3}}{4}\left(\frac{3}{2}\right)^2=\frac{9\sqrt{3}}{16}$$

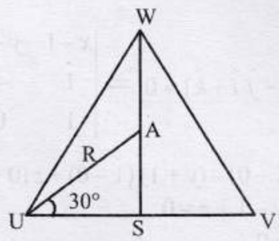
Volume of tetrahedron with coteminous edges

$$\vec{u}, \vec{v}, \vec{w}=\frac{1}{3}(\text{Ar } \Delta UVW)\times OA$$

$$=\frac{1}{3}\times\frac{9\sqrt{3}}{16}\times\frac{1}{2}=\frac{3\sqrt{3}}{32}$$

$\therefore$ Volume of parallelopiped:

$$V=6\times \text{volume of tetrahedron}=\frac{6\times 3\sqrt{3}}{32}=\frac{9\sqrt{3}}{16}$$



Now, $\frac{80}{\sqrt{3}}V=\frac{80}{\sqrt{3}}\times\frac{9\sqrt{3}}{16}=45$

12. **(8)** Let coordinates of P are (a, b, c).

So, coordinates of Q are (0, 0, c) and coordinates of R are (a, b, –c).

Given that, PQ is perpendicular to the plane $x+y=3$.

So, PQ is parallel to the normal of given plane

i.e. $(a\hat{i}+b\hat{j})$ is parallel to $(\hat{i}+\hat{j})$ on comparing

$\Rightarrow a=b$

As mid-point of PQ lies in the plane $x+y=3$, so

$$\frac{a}{2}+\frac{b}{2}=3$$

$\Rightarrow a+b=6 \Rightarrow a=3=b$

Therefore, distance of P from the x-axis

$=\sqrt{b^2+c^2}=5$ (given)

$\Rightarrow b^2+c^2=25$

$\Rightarrow c^2=25-9=16$

$\Rightarrow c=\pm 4$

Hence, $PR=|2c|=8$

13. **(6)** The equation of plane containing the given lines:

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}=0 \Rightarrow x-2y+z=0$$

$\therefore$ Distance between $x-2y+z=0$ and $Ax-2y+z=d$

= Perpendicular distance between parallel planes $(\because A=1)$

$$=\frac{|d|}{\sqrt{6}}=\sqrt{6} \Rightarrow |d|=6.$$

14. **(0.75)**

15. Equation of plane containing vectors $\hat{i}$ and $\hat{i}+\hat{j}$ is

$$\left[\vec{r}-\hat{i}\ \ \hat{i}\ \ \hat{i}+\hat{j}\right]=0 \Rightarrow \begin{vmatrix} x-1 & y & z \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix}=0$$

$\Rightarrow z=0$ ...(i)

Similarly, equation of plane containing vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ is

$$[\hat{r}-(\hat{i}-\hat{j})\ \hat{i}-\hat{j}\ \hat{i}+\hat{k}]=0 \Rightarrow \begin{vmatrix} x-1 & y+1 & z \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix}=0$$

$\Rightarrow (x-1)(-1-0)-(y+1)(1-0)+z(0+1)=0$
$\Rightarrow -x+1-y-1+z=0$
$\Rightarrow x-y+z=0$ ....(ii)

Let $\vec{a}=a\hat{i}+b\hat{j}+c\hat{k}$

Since $\vec{a}$ is parallel to (i) and (ii)
$\therefore c=0$ and $a+b-c=0 \Rightarrow a=-b$

$\therefore$ a vector in direction of $\vec{a}$ is $\hat{i}-\hat{j}$

Let θ is the angle between a and $\hat{i}-2\hat{j}+2\hat{k}$ then

$$\cos\theta=\pm\frac{1.1+(-1)(-2)}{\sqrt{1+1}\sqrt{1+4+4}}=\pm\frac{3}{\sqrt{2}.3}$$

$$\Rightarrow \cos\theta=\pm\frac{1}{\sqrt{2}}$$

$\Rightarrow \theta=\pi/4$ or $3\pi/4$

16. Unit vector perpendicular to plane, $\hat{n}=\pm\frac{\overrightarrow{PQ}\times\overrightarrow{PR}}{|\overrightarrow{PQ}\times\overrightarrow{PR}|}$

$\overrightarrow{PQ}=\hat{i}+\hat{j}-3\hat{k}$; $\overrightarrow{PR}=-\hat{i}+3\hat{j}-\hat{k}$

$$\therefore \overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$$=(-1+9)\hat{i}-(-1-3)\hat{j}+(3+1)\hat{k}=8\hat{i}+4\hat{j}+4\hat{k}$$

$|\overrightarrow{PQ}\times\overrightarrow{PR}|=\sqrt{64+16+16}=\sqrt{96}=4\sqrt{6}$

$$\hat{n}=\pm\left(\frac{8\hat{i}+4\hat{j}+4\hat{k}}{4\sqrt{6}}\right)=\pm\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

17. **(a, b, c)** Let X = (x, y) S: $\{((x-1)^2+(y-2)^2+(z-3)^2)-((x-4)^2+(y-2)^2+(z-7)^2)=50\}$
$\Rightarrow$ S: $\{6x+8z-105=0\}$
Similarly T = $\{6x+8z-5=0\}$
Both S and T represents the equation of plane and parallel to each other.

Other Distance between plane $=\left|\frac{105-5}{\sqrt{36+64}}\right|=10$ unit

So. S will contain a triangle of area 1. So (a) is correct. Hence (b) and (c) are correct but (d) is incorrect.

18. **(a, c)** Equation of line parallel to $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-6}{1}$ through P(1, 3, 2) is $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-2}{1}=\lambda$ (let)

Now, putting any point $(\lambda+1, 2\lambda+3, \lambda+2)$ in plane L_1,
$\lambda+1-2\lambda-3+3(\lambda+2)=6$
$\Rightarrow \lambda=1$
So, point Q (2, 5, 3)
Equation of line through Q (2, 5, 3) perpendicular to L_1 is $\frac{x-2}{1}=\frac{y-5}{-1}=\frac{z-3}{3}=\mu$ (Let)

Putting any point $(\mu+2, -\mu+5, 3\mu+3)$ in plane L_2
$\Rightarrow \mu=-1$
So, point R (1, 6, 0)
(a) PQ $=\sqrt{1+4+1}=\sqrt{6}$
(b) R (1, 6, 0)
(c) Centroid $\left(\frac{4}{3},\frac{14}{3},\frac{5}{3}\right)$
(d) PQ + QR + PR $=\sqrt{6}+\sqrt{11}+\sqrt{13}$

19. **(a, b)** We have planes P_1 and P_2 given as
$P_1: 10x+15y+12z-60=0$ and
$P_2: -2x+5y+4z-20=0$
Thus, equation of pair of planes is
$S: (10x+15y+12z-60)(-2x+5y+4z-20)=0$
Now we will obtain a general point of each line and we will solve it with S. If we get more than one value of variable λ, then the line can be the edge of given tetrahedron.

(a) From option we have $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}$

Let $\frac{x-1}{0}=\frac{y-1}{0}=\frac{z-1}{5}=\lambda$
So, point is $(1, 1, 5\lambda+1)$
So, $(60\lambda-23)(20\lambda-17)=0$
$\lambda=\frac{23}{60}$ and $\frac{17}{20}$
So, it can be the edge of tetrahedron.
(b) Similarly for option (b)
point is $(-5\lambda+6, 2\lambda, 3\lambda)$
So, $(16\lambda)(32\lambda-32)=0$
$\Rightarrow \lambda=0$ and 1
So, it can be the edge of tetrahedron.
(c) Similarly for option (c)
Point is $(-2\lambda, 5\lambda+4, 4\lambda)$
So, $(103\lambda)(45\lambda)=0$
$\lambda=0$ only
So, it cannot be the edge of tetrahedron.

(d) Similarly for option (d)
Point is $(\lambda, -2\lambda+4, 3\lambda)$

$\Rightarrow (16\lambda)(-2\lambda)=0$

$\Rightarrow \lambda=0$ only

Hence, it cannot be the edge of tetrahedron.

20. **(a, b, c)** We are given that equation of plane is

$\vec{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$

This can be written as

$\vec{r} = \hat{k} + t(-\hat{i}+\hat{j}) + p(-\hat{i}+\hat{k})$

Now, equation of plane in standard form is

$\left[\vec{r}-\hat{k} \;\; -\hat{i}+\hat{j} \;\; -\hat{i}+\hat{k}\right]=0$

$\therefore \quad x+y+z=1$ ...(i)

Coordinate of $Q=(10, 15, 20)$

Coordinate of $S=(\alpha, \beta, \gamma)$

$$\therefore \quad \frac{\alpha-10}{1}=\frac{\beta-15}{1}=\frac{\gamma-20}{1}=\frac{-2(10+15+20-1)}{3}$$

$$\left[\because \text{ point of reflection is given as } \frac{x-x_1}{a} = \frac{y-y_1}{b}=\frac{z-z_1}{c}=\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}\right]$$

$$\therefore \quad \alpha-10=\beta-15=\gamma-20=-\frac{88}{3}$$

$$\therefore \quad \alpha=-\frac{58}{3}, \beta=-\frac{43}{3}, \gamma=-\frac{28}{3}$$

$\therefore \quad 3(\alpha+\beta)=-101, 3(\beta+\gamma)=-71$

$3(\gamma+\alpha)=-86$ and $3(\alpha+\beta+\gamma)=-129$

21. **(a, b)** The point of intersection of L_1 and L_2 is (1, 0, 1)

$\because$ Line L passes through the point of intersection (1, 0, 1) of L_1 and L_2

$$\therefore \frac{1-\alpha}{\ell}=-\frac{1}{m}=\frac{1-\gamma}{-2} \quad \text{...(i)}$$

$\because$ Line L_1 bisects the acute angle between the lines L_1 and L_2, then

$$\vec{r}=\hat{i}+\hat{k}+\lambda\left(\frac{\hat{i}-\hat{j}+3\hat{k}-3\hat{i}-\hat{j}+\hat{k}}{\sqrt{11}}\right)$$

$$\Rightarrow \vec{r}=\hat{i}+\hat{k}+t(\hat{i}+\hat{j}-2\hat{k})$$

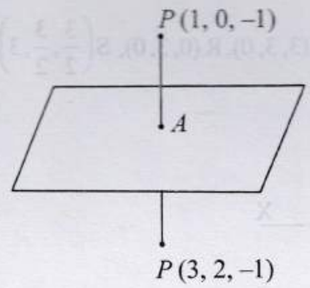
$$\Rightarrow \frac{\ell}{1}=\frac{m}{1}=\frac{-2}{-2} \Rightarrow \ell=m=1$$

From (i), $\frac{1-\alpha}{1}=-1 \Rightarrow \alpha=2$

and $\frac{1-\gamma}{-2}=-1 \Rightarrow \gamma=-1$

$\therefore \alpha-\gamma=2-(-1)=3$ and $l+m=1+1=2$

22. **(a, b, c)**



Mid-point of PQ = A (2, 1, –1)

D.r's of PQ = 2, 2, 0

Since PQ perpendicular to plane and mid-point lies on plane

$\therefore$ Equation of plane :

$2(x-2)+2(y-1)+0(z+1)=0$

$\Rightarrow x-2+y-1=0$

$\Rightarrow x+y=3$ comparing with $\alpha x+\beta y+\gamma z=\delta$,

we get $\alpha=1, \beta=1, \gamma=0$ and $\delta=3$.

$\therefore$ option (a), (b), (c) are true.

23. **(c, d)**

(a) Direction vector of line of intersection of two planes will be given by $\vec{n}_1 \times \vec{n}_2$.

$$\therefore \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\hat{i}-3\hat{j}+3\hat{k}$$

$\therefore$ dr's of line of intersection of P_1 and P_2 are 1, – 1, 1

$\therefore$ (a) is not correct.

(b) The standard form of given line as

$$\frac{x-\frac{4}{3}}{3}=\frac{y-\frac{1}{3}}{-3}=\frac{z}{3}$$

$\because 1\times 3+(-1)(-3)+1(3)=9\neq 0$

$\therefore$ This line is not perpendicular to line of intersection

$\therefore$ (b) is not correct.

(c) Let θ be the angle between P_1 and P_2 then

$$\cos\theta = \left|\frac{2\times1+1\times2+(-1)\times1}{\sqrt{6}\sqrt{6}}\right| = \frac{3}{6}=\frac{1}{2}$$

$\therefore \theta = 60°$

Hence (c) is correct.

(d) Equation of plane P_3 :

$1(x-4)-1(y-2)+1(z+2)=0$

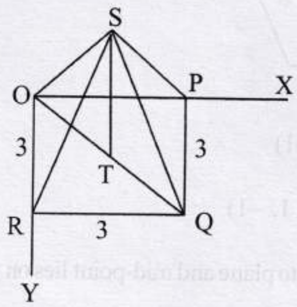
$\Rightarrow x-y+z=0$

Distance of (2, 1, 1) from $P_3 = \frac{2-1+1}{\sqrt{1+1+1}}=\frac{2}{\sqrt{3}}$

$\therefore$ (d) is correct.

24. **(b, c, d)** According to question the coordinates of vertices of pyramid OPQRS will be

O(0, 0, 0), P (3, 0, 0), Q (3, 3, 0), R (0, 3, 0), S$\left(\frac{3}{2}, \frac{3}{2}, 3\right)$



dr's of OQ = 1, 1, 0
dr's of OS = 1, 1, 2
$\therefore$ acute angle between OQ and OS

$$= \cos^{-1}\left(\frac{2}{\sqrt{2}\times\sqrt{6}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \neq \frac{\pi}{3}$$

$\therefore$ (a) is not correct

$$\text{Eqn of plane OQS} = \begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2x - 2y = 0$ or $x - y = 0$

$\therefore$ (b) is correct.

length of perpendicular from P (3, 0, 0) to plane $x - y = 0$ is =

$$\left|\frac{3-0}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$$

$\therefore$ (c) is correct.

$$\text{Eqn of RS}: \frac{x}{\frac{3}{2}} = \frac{y-3}{\frac{-3}{2}} = \frac{z}{3} \text{ or } \frac{x}{1} = \frac{y-3}{-1} = \frac{z}{2} = \lambda$$

$\therefore$ Any point ON RS is N $(\lambda, -\lambda+3, 2\lambda)$

Since ON is perpendicular to RS,

$\because$ ON $\perp$ RS $\Rightarrow 1\times\lambda - 1(-\lambda+3) + 2\times 2\lambda = 0$

$$\Rightarrow \lambda = \frac{1}{2} \Rightarrow N\left(\frac{1}{2}, \frac{5}{2}, 1\right)$$

$$\therefore \text{ON} = \sqrt{\frac{1}{4} + \frac{25}{4} + 1} = \sqrt{\frac{15}{2}}$$

$\therefore$ (d) is correct

25. **(a, b)** $\because$ All the points on L are at a constant distance from P_1 and P_2 that means L is parallel to both P_1 and P_2

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\therefore \quad L: \frac{x}{1} = \frac{y}{-3} = \frac{z}{-5} = \lambda \text{ (say)}$$

$\therefore$ Any point on line L is $(\lambda, -3\lambda, -5\lambda)$

Equation of line perpendicular to P_1 drawn from any point on L is

$$\frac{x-\lambda}{1} = \frac{y+3\lambda}{2} = \frac{z+5\lambda}{-1} = \mu$$

$\therefore \quad M(\mu+\lambda, 2\mu-3\lambda, -\mu-5\lambda)$

But M lies on P_1 so, it satisfy the eqn. of P_1.

$$\therefore \quad \mu+\lambda+4\mu-6\lambda+\mu+5\lambda+1=0 \Rightarrow \mu = \frac{-1}{6}$$

$$\therefore \quad M\left(\lambda-\frac{1}{6}, -3\lambda\frac{-1}{3}, -5\lambda+\frac{1}{6}\right)$$

For locus of M,

$$x = \lambda - \frac{1}{6}, y = -3\lambda - \frac{1}{3}, z = 5\lambda + \frac{1}{6}$$

$$\Rightarrow \quad \frac{x+1/6}{1} = \frac{y+1/3}{-3} = \frac{z-1/6}{-5} = \lambda$$

On checking the given point, we find $\left(0, \frac{-5}{6}, \frac{-2}{3}\right)$ and $\left(\frac{-1}{6}, \frac{-1}{3}, \frac{1}{6}\right)$ satisfy the above eqn.

26. **(b, d)** $P_3: (x+z-1) + \lambda y = 0 \Rightarrow x + \lambda y + z - 1 = 0$

Distance of point (0, 1, 0) from P_3:

$$\left|\frac{\lambda-1}{\sqrt{2+\lambda^2}}\right| = 1 \Rightarrow \lambda^2 - 2\lambda + 1 = \lambda^2 + 2 \Rightarrow \lambda = \frac{-1}{2}$$

Distance of point (α, β, γ) from P_3:

$$\left|\frac{\alpha+\lambda\beta+\gamma-1}{\sqrt{2+\lambda^2}}\right| = 2 \Rightarrow \frac{\alpha - \frac{1}{2}\beta + \gamma - 1}{\frac{3}{2}} = \pm 2$$

$$\Rightarrow \quad \alpha - \frac{1}{2}\beta + \gamma - 1 = \pm 3 \Rightarrow 2\alpha - \beta + 2\gamma - 2 = \pm 6$$

$$\Rightarrow \quad 2\alpha - \beta + 2\gamma - 8 = 0 \text{ or } 2\alpha - \beta + 2\gamma + 4 = 0$$

27. **(a, d)** Given that L_1 and L_2 are coplanar, therefore

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow (5-\alpha)[6-5\alpha+\alpha^2-2]=0$

$\Rightarrow (5-\alpha)(\alpha-1)(\alpha-4)=0 \Rightarrow \alpha=1,4,5.$

28. **(b, c)** Given that lines are coplanar.

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \Rightarrow k=\pm 2$$

For $k=2$, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0 \Rightarrow y-z+1=0$$

For $k=-2$, equation of the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0 \Rightarrow y+z+1=0$$

29. **(b, d)** Normal vector of plane P_1 is

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 3 \\ 0 & 4 & -3 \end{vmatrix} = -18\hat{i}$$

Normal vector of plane P_2 is

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 3 & 3 & 0 \end{vmatrix} = 3\hat{i}-3\hat{j}-3\hat{k}$$

$\therefore \vec{A}$ is parallel to $\pm(\vec{n}_1 \times \vec{n}_2) = \pm(-54\hat{j}+54\hat{k})$

Now, angle between $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is given by

$$\cos\theta = \pm\frac{(-54\hat{j}+54\hat{k}).(2\hat{i}+\hat{j}-2\hat{k})}{54\sqrt{2}.3} = \pm\frac{1}{\sqrt{2}}$$

$\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

30. **(b)** For largest possible distance between plane H_o and l_2, the line l_2 must be parallel to plane H_o.

$\therefore H_o$ will be the plane containing the line l_1 and parallel to l_2

Normal vector $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}-\hat{k}$

$\therefore H_o : x-z = c/(0,0,0) \Rightarrow c=0$

$\therefore H_o : x-z=0$

(P) Distance of point (0, 1, –1) from H_o.

$$d(H_o) = \left|\frac{0-(1)}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$$

(Q) The distance of the point (0, 1, 2) from $H_o = \left|\frac{0-2}{\sqrt{2}}\right| = \sqrt{2}$

(R) The distance of origin from $H_o = \left|\frac{0}{\sqrt{2}}\right| = 0$

(S) Point of intersection of planes $y=z$, $x=1$ and H_o is (1, 1, 1).

Distance $=\sqrt{1+1+1}=\sqrt{3}.$

31. **(a)** Let any point on L_1 is $(2\lambda+1, -\lambda, \lambda-3)$

and that on L_2 is $(\mu+4, \mu-3, 2\mu-3)$

For point of intersection of L_1 and L_2

$2\lambda+1=\mu+4, -\lambda=\mu-3, \lambda-3=2\mu-3$

$\Rightarrow \lambda=2, \mu=1$

$\therefore$ Intersection point of L_1 and L_2 is (5, –2, –1)

Equation of plane passing through, (5, –2, –1) and perpendicular to P_1 & P_2 is given by

$$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$$

$\Rightarrow x-3y-2z=13$

$\therefore a=1, b=-3, c=-2, d=13$

or (P) $\rightarrow$(3) (Q) $\rightarrow$(2) (R) $\rightarrow$(4) (S) $\rightarrow$(1)

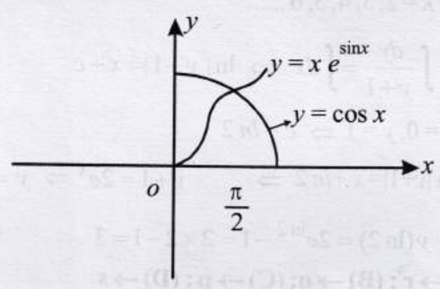
32. **A $\rightarrow$ p; B $\rightarrow$ q,s; C $\rightarrow$ q,r,s,t; D $\rightarrow$ r**

(A) Let us consider two functions

$y = xe^{\sin x}$ and $y=\cos x$

The range of $y=xe^{\sin x}$ is $\left(0, \frac{\pi e}{2}\right)$ and

$\frac{dy}{dx} = e^{\sin x} + xe^{\sin x}\cos x \geq 0$, for $x \leftarrow \left(0, \frac{\pi}{2}\right)$, so, it is an increasing function on $\left(0, \frac{\pi}{2}\right)$. Their graph are as shown in the figure below :



Clearly the two curves meet only at one point, therefore the given equation has only one solution in $\left(0, \frac{\pi}{2}\right)$.

(B) Since given planes intersect in a straight-line

$$\therefore \begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad k(k-4)-4(4-4)+1(8-2k)=0$

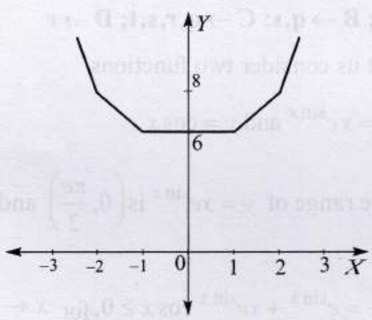
$\Rightarrow \quad k^2-6k+8=0 \Rightarrow (k-2)(k-4)=0$

$\Rightarrow \quad k = 2 \text{ or } 4$

(C) We have $f(x)=|x-1|+|x-2|+|x+1|+|x+2|$

$$= \begin{cases} -4x & , \ x \le -2 \\ -2x+4, & -2 < x \le -1 \\ 6, & -1 < x \le 1 \\ 2x+4, & 1 < x \le 2 \\ 4x, & x \ge 2 \end{cases} \qquad \left[\because |x-1| \begin{cases} x-1, \text{is } x \ge 1 \\ -(x-1) \text{ is } x < 1 \end{cases}\right]$$

The graph of the above function is as given below



Clearly, from graph, $f(x) \ge 6$

$\Rightarrow \quad 4k \ge 6 \quad \Rightarrow \quad k \ge \frac{3}{2}$

$\therefore \quad k = 2, 3, 4, 5, 6, ...$

(D) $\int \frac{dy}{y+1} = \int dx \quad \Rightarrow \ln|y+1| = x + c$

At $x = 0, y = 1 \Rightarrow c = \ln 2$

$\therefore \ln|y+1| = x + \ln 2 \Rightarrow \quad y+1 = 2e^x \Rightarrow y = 2e^x - 1$

$\therefore \quad y(\ln 2) = 2e^{\ln 2} - 1 = 2 \times 2 - 1 = 3$

33. **(A) → r ; (B) → q; (C) → p ; (D) → s**

The determinant of the coefficient matrix of given equation, as

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

(A) When $a+b+c \ne 0$ and

$a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \quad (a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow \quad a=b=c \quad (\text{but} \ne 0 \text{ as } a+b+c \ne 0)$

This equation represent identical planes.

(B) When $a+b+c=0$ and

$a^2+b^2+c^2-ab-bc-ca \ne 0$

$\Rightarrow \quad \Delta = 0$ and a, b, c are not all equal.

$\therefore$ All equations are not identical but have infinite many solutions.

$\therefore \quad ax+by=(a+b)z$ ... (i) (using $a+b+c=0$)

and $bx+cy=(b+c)z$ ... (ii)

On Solving eqn. (i) and (ii) we, get

$\Rightarrow \quad (b^2-ac)y=(b^2-ac)z \Rightarrow y=z$

$\Rightarrow \quad ax+by+cy=0 \Rightarrow ax=ay \Rightarrow x=y$

$\Rightarrow \quad x=y=z$

$\therefore$ The equations represent the line $x=y=z$

(C) When $a+b+c \ne 0$ and

$a^2+b^2+c^2-ab-bc-ca \ne 0$

$\Rightarrow \quad \Delta \ne 0 \Rightarrow$ Equations have only trivial solution i.e., $x=y=z=0$

$\therefore$ the equations represents the three planes meeting at a single point namely origin.

(D) When $a+b+c=0$ and

$a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \quad a=b=c$ and $\Delta=0 \Rightarrow a=b=c=0$

$\Rightarrow$ All equations are satisfied by all x, y, and z.

$\Rightarrow$ The equations represent the whole of the three dimensional space (all points in 3–D)

34. **(A) → (s); (B) → (p); (C) → (q), (r); (D) → (s)**

(A) $\quad x+y=|a|$

$\quad ax-y=1$

$\quad \overline{(1+a)x=1+|a|}$

$\Rightarrow x = \frac{1+|a|}{a+1} \Rightarrow y = \frac{a|a|-1}{a+1}$

$\because$ Rays intersect each other in I quad i.e. $x > 0, y \ge 0$

$\Rightarrow a+1>0$ and $a|a|-1>0 \Rightarrow a>1$

$\therefore a_0 = 1$ (A) → (s)

(B) Given that (α, β, γ) lies on the plane $x+y+z=2$

$\Rightarrow \alpha+\beta+\gamma=2$

Also $\hat{k}\times(\hat{k}\times\vec{a})=(\hat{k}.\vec{a})\hat{k}-(\hat{k}.\hat{k})\vec{a}$

$\Rightarrow \gamma\hat{k}-\alpha\hat{i}-\beta\hat{j}-\gamma\hat{k}=0 \Rightarrow \alpha\hat{i}+\beta\hat{j}=0$

$\Rightarrow \alpha=0=\beta \Rightarrow \gamma=2 \qquad (\because \alpha+\beta+\gamma=2)$

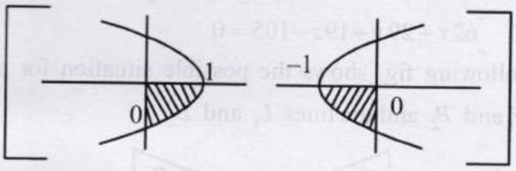
(B) $\to$ (p)

(C) $\left|\int_0^1(1-y^2)dy\right|+\left|\int_0^1(y^2-1)dy\right|$

$=2\left|\int_0^1(1-y^2)dy\right|=\frac{4}{3}$

$\because y=\sqrt{1-x}, \Rightarrow y^2=-(x-1)$ and $y=\sqrt{1+x}$

$\Rightarrow y^2=(x+1)$ It is clear from above figure that



$\left|\int_0^1\sqrt{1-x}\,dx\right|+\left|\int_{-1}^0\sqrt{1+x}\,dx\right| \quad =2\int_0^1\sqrt{1-x}\,dx$

$=2\int_0^1\sqrt{x}\,dx \quad \left[\text{Using}\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$=\left[2.\frac{2}{3}x^{3/2}\right]_0^1=\frac{4}{3}, \qquad$ (C) $\to$ (r) and (q)

(D) Given that $\sin A\sin B\sin C+\cos A\cos B=1$

We know that $\sin A\sin B\sin C+\cos A\cos B\le\sin A\sin B+\cos A\cos B=\cos(A-B)$

$\Rightarrow \cos(A-B)\ge 1 \Rightarrow \cos(A-B)=1$

$\Rightarrow A-B=0 \Rightarrow A=B$

$\therefore$ Given relation becomes $\sin^2 A\sin C+\cos^2 A=1$

$\Rightarrow \sin C=1,$

(D) $\to$ (s)

35. **(b)** Vector in the direction of $L_1=\vec{b_1}=3\hat{i}+\hat{j}+2\hat{k}$

Vector in the direction of $L_2=\vec{b_2}=\hat{i}+2\hat{j}+3\hat{k}$

$\therefore$ Vector perpendicular to both L_1 and L_2

$$=\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&1&2\\1&2&3\end{vmatrix}=-\hat{i}-7\hat{j}+5\hat{k}$$

$\therefore$ Required unit vector

$$=\hat{b}=\frac{-\hat{i}-7\hat{j}+5\hat{k}}{\sqrt{1+49+25}}=\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$$

36. **(d)** The shortest distance between L_1 and L_2 is

$$=\frac{(\vec{a}_2-\vec{a}_1).\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}=(\vec{a}_2-\vec{a}_1).\hat{b}$$

Since, $a_1=-\hat{i}-2\hat{j}-\hat{k} \qquad a_2=2\hat{i}-2\hat{j}+3\hat{k}$

$\therefore \vec{a}_2-\vec{a}_1=3\hat{i}+4\hat{k} \qquad \therefore (\vec{a}_2-\vec{a}_1).\hat{b}$

$$\therefore (3\hat{i}+4\hat{k}).\left(\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}\right)=\frac{-3+20}{5\sqrt{3}}=\frac{17}{5\sqrt{3}}$$

37. **(c)** The plane passing through $(-1,-2,-1)$ and having normal along $\vec{b}$ is

$-1(x+1)-7(y+2)+5(z+1)=0$

$\Rightarrow x+7y-5z+10=0$

$\therefore$ Distance of point $(1,1,1)$ from the above plane is

$$=\frac{1+7\times1-5\times1+10}{\sqrt{1+49+25}}=\frac{13}{\sqrt{75}}$$

38. **(d)** The given planes are

$P_1: x-y+z=1 \qquad$ ...(1)

$P_2: x+y-z=-1 \qquad$ ...(2)

$P_3: x-3y+3z=2 \qquad$ ...(3)

Since, line L_1 is intersection of planes P_2 and P_3.

$\therefore L_1$ is parallel to the vector

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-1\\1&-3&3\end{vmatrix}=-4\hat{j}-4\hat{k}$$

Line L_2 is intersection of P_3 and P_1

$\therefore L_2$ is parallel to the vector

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&1\\1&-3&3\end{vmatrix}=-2\hat{j}-2\hat{k}$$

And line L_3 is intersection of P_1 and P_2

$\therefore L_3$ is parallel to the vector

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&1\\1&1&-1\end{vmatrix}=2\hat{j}+2\hat{k}$$

Clearly lines L_1, L_2 and L_3 are parallel to each other.

$\therefore$ Statement–1 is False

Also family of planes passing through the intersection of P_1 and P_2 is $P_1+\lambda P_2=0$.

$\Rightarrow x(1+\lambda)+y(\lambda-1)+z(1-\lambda)+(\lambda-1)=0$

The three planes have a common point

$$\frac{1+\lambda}{1}=\frac{\lambda-1}{-3}=\frac{1-\lambda}{3}=\frac{1-\lambda}{2} \quad ...(i)$$

Taking $\frac{1+\lambda}{1}=\frac{1-\lambda}{2}$, we get $\lambda=-\frac{1}{3}$ and taking $\frac{1+\lambda}{1}=\frac{1-\lambda}{3}$, we get $\lambda=-\frac{2}{3}$.

$\therefore$ There is no value of λ which satisfies eq (i).

$\therefore$ The three planes do not have a common point.

$\Rightarrow$ Statement 2 is true.

$\therefore$ (d) is the correct option.

39. **(d)** The line of intersection of given plane is

$3x-6y-2z-15=0=2x+y-2z-5$

For $z=0$, we get $x=3$ and $y=-1$

$\therefore$ Line passes through $(3,-1,0)$.

Direction vector of line is

$$\vec{b}=\vec{x}_1\times\vec{x}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -6 & -2\\ 2 & 1 & -2\end{vmatrix}$$

$$=14\hat{i}+2\hat{j}+15\hat{k}$$

$\therefore$ Eqn. of line is $\frac{x-3}{14}=\frac{y+1}{2}=\frac{z}{15}=t$

whose parametric form is

$x=3+14t, y=2t-1, z=15t$

$\therefore$ Statement-I is false

$\therefore$ Statement 2 is true.

40. Equation of plane containing line of intersection of two given planes is given by

$$(2x-y+z-3)+\lambda(3x+y+z-5)=0$$

$$\Rightarrow (3\lambda+2)x+(\lambda-1)y+(\lambda+1)z+(-5\lambda-3)=0$$

since distance of this plane from the pt. $(2,1,-1)$ is $\frac{1}{\sqrt{6}}$

$$\therefore \left|\frac{(3\lambda+2)2+(\lambda-1)1+(\lambda+1)(-1)+(-5\lambda-3)}{\sqrt{(3\lambda+2)^2+(\lambda-1)^2+(\lambda+1)^2}}\right|=\frac{1}{\sqrt{6}}$$

$$\Rightarrow \left|\frac{\lambda-1}{\sqrt{11\lambda^2+12\lambda+6}}\right|=\frac{1}{\sqrt{6}}$$

Squaring both sides, we get

$$\frac{(\lambda-1)^2}{11\lambda^2+12\lambda+6}=\frac{1}{6}$$

$$\Rightarrow 6\lambda^2-12\lambda+6-11\lambda^2-12\lambda-6=0$$

$$\Rightarrow 5\lambda^2+24\lambda=0 \Rightarrow \lambda(5\lambda+24)=0$$

$$\Rightarrow \lambda=0 \text{ or } -24/5$$

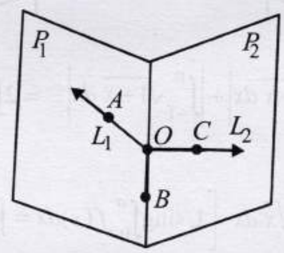
$\therefore$ The required equations of planes are

$2x-y+z-3=0$

or $\left[3\left(\frac{-24}{5}\right)+2\right]x+\left[-\frac{24}{5}-1\right]y+\left[-\frac{24}{5}+1\right]z-5\left(\frac{-24}{5}\right)-3=0$

or $62x+29y+19z-105=0$

41. Following fig. shows the possible situation for planes P_1 and P_2 and the lines L_1 and L_2



A corresponds to one of A', B', C' and B corresponds to one of the remaining of A', B', C' and C corresponds to third of A', B', C'.

Hence six such permutations are possible

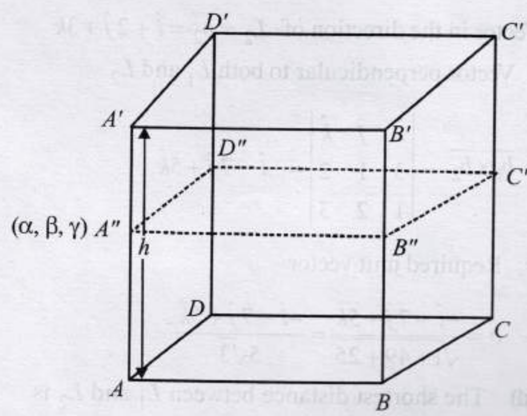
e.g., One of the permutations may A = A', B = B', C = C'

From the given conditions : A lies on L_1, B lies on the line of intersection of P_1 and P_2 and 'C' lies on the line L_2 on the plane P_2.

Now, A' lies on L_2 = C, B' lies on the line of intersection of P_1 and P_2 = B and C' lie on L_1 on plane P_1 = A.

Hence there exist a particular set [A', B', C'] which is the permutation of [A, B, C] such that both (i) and (ii) is satisfied. Here [A', B', C'] ≡ [C, B, A].

42.



Let

Let equation of plane $ABCD$ be $ax+by+cz+d=0$, h be the height of original parallelepiped S. and $A''(\alpha,\beta,\gamma)$

Then height of new parallelepipe dT is the length of perpendicular from A'' to $ABCD$

i.e. $\frac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}$

$V_T=\frac{90}{100}V_s$

$\therefore \quad (ar\,ABCD)\times\frac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}$

$=(ar\,ABCD)\times h\times 0.9$

But given that,

$\frac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}=0.9h$

$\Rightarrow \quad a\alpha+b\beta+c\gamma+(d-0.9h\sqrt{a^2+b^2+c^2})=0$

$\therefore$ Locus of $A''(\alpha,\beta,\gamma)$ is

$ax+by+cz+(d-0.9h\sqrt{a^2+b^2+c^2})=0$

which is a plane parallel to $ABCD$. Hence proved.

43. Equation of plane through (1, 1, 1) is

$$\begin{vmatrix}x-1 & y-1 & z-1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{vmatrix}=0$$

$\Rightarrow \quad (x-1)(0-1)-(y-1)(0+1)+(z-1)(-1-0)=0$

$\Rightarrow \quad -1(x-1)-1(y-1)-1(z-1)=0 \Rightarrow x+y+z=3$

$\Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{3}=1 \quad$ ...(1)

$\therefore$ plane intersect the axes at

$A(3,0,0), B(0,3,0)$, and $C(0,0,3)$

$\therefore$ Vol.of tetrahedron $OABC$

$=\frac{1}{6}\times$ Area of base $\times$ altitude

$=\frac{1}{6}\times \text{Ar}(\Delta ABC)\times$ length of $\perp^{\text{lar}}$ (0,0,0) to plane (1)

$=\frac{1}{6}\times\frac{1}{2}\left[\frac{\sqrt{3}}{4}\times|\overline{AB}|^2\right]\times\left[\left|\frac{-3}{\sqrt{1+1+1}}\right|\right]$

($\because \Delta ABC$ is an equilateral triangle)

$=\frac{1}{12}\times\frac{\sqrt{3}}{4}\times(3\sqrt{2})^2\times\sqrt{3}=\frac{3\times18}{48}=\frac{9}{2}$ cubic units.

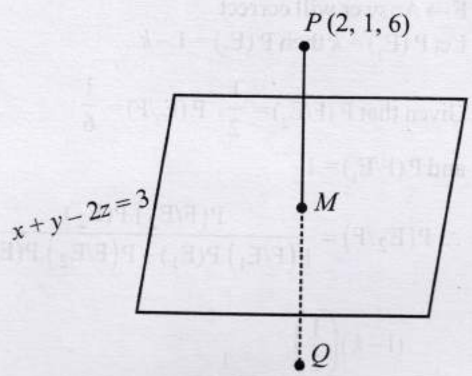
44. (i) Equation of plane passing through (2, 1, 0), (5, 0, 1) and (4, 1, 1) is

$$\begin{vmatrix}x-2 & y-1 & z-0\\ 5-2 & 0-1 & 1-0\\ 4-2 & 1-1 & 1-0\end{vmatrix}=0 \Rightarrow \begin{vmatrix}x-2 & y-1 & z\\ 3 & -1 & 1\\ 2 & 0 & 1\end{vmatrix}=0$$

$\Rightarrow \quad (x-2)(-1-0)-(y-1)(3-2)+z(0-(-2))=0$

$\Rightarrow \quad -x+2-y+1+2z=0 \quad \Rightarrow \quad x+y-2z=3$

(ii)



Eq^n of PQ passing through $P(2, 1, 6)$ and $\perp$ to plane $x+y-2z=3$, is given by

$\frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\lambda$

$\therefore \quad Q(\lambda+2,\lambda+1,-2\lambda+6)$

$\therefore \quad$ Mid. pt. of PQ

i.e. $\quad M\left(\frac{2+\lambda+2}{2},\frac{1+\lambda+1}{2},\frac{6-2\lambda+6}{2}\right)$

$=\left(\frac{\lambda+4}{2},\frac{\lambda+2}{2},\frac{12-2\lambda}{2}\right)$

But M lies on plane $x+y-2z=3$

$\therefore \quad \frac{\lambda+4}{2}+\frac{\lambda+2}{2}-(12-2\lambda)=3$

$\Rightarrow \lambda+4+\lambda+2-24+4\lambda=6 \Rightarrow 6\lambda=24 \Rightarrow \lambda=4$

$\therefore \quad Q(4+2,4+1,-8+6)=(6,5,-2)$